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CP violation arising from particle-antiparticle mixing

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ABSTRACT Indirect CP violation arising from particle-antiparticle mixing is calculated from the box diagrams in the Standard Model for K^o–$\bar{K}^o$, B^o–$\bar{B}^o$ and B_s^o–$\bar{B}_s^o$ systems. The CP violation parameter for each of the systems is shown to be closely related to the relative phases of the Kobayashi-Maskawa matrix elements.

ABSTRAK Perlanggaran CP taklangsung yang berhasil dari percampuran zarah-antizarah dikira dari rajah-rajah kotak di dalam Model Piawai untuk sistem-sistem K^o–$\bar{K}^o$, B^o–$\bar{B}^o$ dan B_s^o–$\bar{B}_s^o$. Parameter perlanggaran CP bagi setiap sistem itu ditunjukkan berhubung rapat dengan fasa-fasa relatif di antara unsur-unsur matriks Kobayashi-Maskawa.

(CP violation, particle-antiparticle mixing, Standard Model, Kobayashi-Maskawa matrix)

INTRODUCTION

Large particle-antiparticle mixing is observed in K^o–$\bar{K}^o$, B^o–$\bar{B}^o$ and B_s^o–$\bar{B}_s^o$ systems. In the case of K^o–$\bar{K}^o$ system, such a mixing gives rise to two distinct mass eigenstates, $\bar{K}_S^o$ and $\bar{K}_L^o$, with decay lifetimes of 0.89310^{-10} s and 5.1710^{-8} s respectively, and a mass difference of [1]

$$\Delta m(K) = m(K_L) - m(K_S) = 3.51 \times 10^{-12}\,\text{MeV} \qquad (1)$$

Mixing in the B^o–$\bar{B}^o$ system is measured by the mixing parameter $\chi(B)$ [1]

$$\chi(B) = \Gamma(B \to \mu^- X)/\Gamma(B \to \mu^\pm X)$$
$$= 0.156 \pm 0.024 \qquad (2)$$

The two mass eigenstates, B_H^o and B_L^o, have a mass difference of [1]

$$\Delta m(B) = (3.4 \pm 0.4)\,\text{MeV} \qquad (3)$$

but do not have noticeably distinct decay lifetimes.

The B_s^o–$\bar{B}_s^o$ system is also observed to have large mixing, with a mixing parameter of [1]

$$\chi(B_s) = 0.62 \pm 0.13 \qquad (4)$$

The two mass eigenstates arising from mixing have a mass difference of [1]

$$\Delta m(B_s) > 1.2 \times 10^{-9}\,\text{MeV} \qquad (5)$$

but, again, do not differ noticeably in the lifetimes.

Within the Standard Model, particle-antiparticle mixing arises from higher order weak interactions, the main contributions of which come from the box diagrams of Fig.1 [2]. Depicted in Fig.1 are the Feynman diagrams that give rise to K^o–$\bar{K}^o$ mixing. Feynman diagrams that contribute to B^o–$\bar{B}^o$ mixing and B_s^o–$\bar{B}_s^o$ mixing are obtained by replacing respectively the external s-quark and d-quark by the b-quark. The internal quark lines, i and j, can be a u-, c- or t-quark.

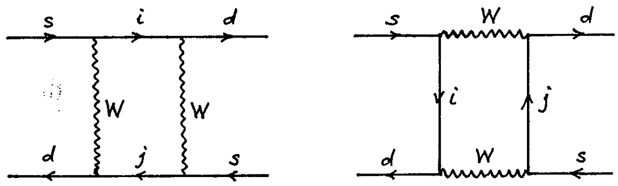

Figure 1. Box diagrams within the Standard Model that give rise to $\bar{K}^o$–K^o mixing. The internal quark lines i, j, can be a u, c or t quark. B^o–$\bar{B}^o$ (B_s^o–$\bar{B}_s^o$) mixing is described by similar diagrams with the external s (d) quark lines replaced by b quark.

Weak interactions of the quarks are described, in the Standard Model, by the following Lagrangian:

$$\mathcal{L}_{(quark)} = \frac{ig}{\sqrt{2}}(\bar{u},\bar{c},\bar{t})\gamma^\mu LV\begin{pmatrix} d \\ s \\ b \end{pmatrix}W_\mu + c.c. \qquad (6)$$

where V is the Kobayashi-Maskawa (K-M) mixing matrix [3]. The magnitudes of the K-M matrix elements are approximately given by

$$|V| \sim \begin{pmatrix} 1 & 0.22 & 0.003 \\ 0.22 & 1 & 0.04 \\ 0.01 & 0.04 & 1 \end{pmatrix} \qquad (7)$$

For three families of quarks, the K-M matrix contains a complex phase which gives rise to CP violation effects in a natural way. Because of this complex phase, the box diagrams of Fig.1 provide a definite connection between particle-antiparticle mixing and CP violation in such a system, the so-called *indirect* CP violation.

In this paper, I shall exploit the box diagrams to derive definite relationship between relative phase among the different K-M matrix on the one-hand, and the CP violation parameter on the other.

DESCRIPTION OF CP VIOLATION

In this section, I shall make specific reference to K^o–$\bar{K}^o$ mixing as a generic case for the three particle-antiparticle systems. The box diagrams of Fig.1 give rise to $\Delta S = 2$ effective Hamiltonian $\mathcal{H}(\Delta S = 2)$, and hence to off-diagonal element of the K^o–$\bar{K}^o$ mass matrix

$$\langle K^o | \mathcal{H}(\Delta S = 2) | \bar{K}^o \rangle = M_{12} - i\Gamma_{12}/2 \qquad (8)$$

where M_{12} and Γ_{12} are respectively the dipersive and absorptive parts of the off-diagonal element of the mass matrix.

Diagonalizing the mass matrix gives two distinct mass eigenstates, which can be written in the following form:

$$|K_{L,S}\rangle = [2(1 + |\varepsilon|^2)]^{-1/2} \, [(1 + \varepsilon)\, |K^o\rangle + (1 - \varepsilon)|K^o\rangle] \qquad (9)$$

where ε is the *indirect* CP violation parameter.

As CP violation is a small effect, we have

$$\mathrm{Im}M_{12} << \mathrm{Re}M_{12}, \quad \mathrm{Im}\Gamma_{12} << \mathrm{Re}\Gamma_{12}, \quad \mathrm{Im}\Gamma_{12} << \mathrm{Im}M_{12} \qquad (10)$$

This greatly simplifies the expressions for K_L–K_S mass difference Δm, their decay rate difference $\Delta\Gamma$, and the indirect CP violation parameter ε:

$$\Delta m \approx \mathrm{Re}M_{12} \qquad (11)$$

$$\Delta\Gamma \approx 2\mathrm{Re}\Gamma_{12} \qquad (12)$$

$$\varepsilon \approx (i/2)\, \frac{\mathrm{Im}\, M_{12} - i\, \mathrm{Im}\, \Gamma_{12}/2}{\mathrm{Re}\, M_{12} - i\, \mathrm{Re}\, \Gamma_{12}/2} \qquad (13)$$

In the next section, I shall give the explicit result for the dispersive part, M_{12}, of the off-diagonal mass matrix element from the box diagrams. The absorptive part, Γ_{12}, will be deduced from a knowledge of $\Delta\Gamma$ and Δm.

EXPLICIT RESULT FROM THE BOX DIAGRAMS

The calculation of M_{12} from the box diagrams is straightforward. A detailed calculation gives

$$\mathcal{H}_{eff} = \frac{G_F^2 M_W^2}{4\pi^2} \mathcal{O}^{\Delta S=2} \{ \lambda_u^2 \bar{E}(x_u) + \lambda_c^2 \bar{E}(x_c) + \lambda_t^2 \bar{E}(x_t)$$
$$+ \lambda_u \lambda_c E(x_u, x_c) + \lambda_u \lambda_t E(x_u, x_t) + \lambda_c \lambda_t E(x_c, x_t) \} \qquad (14)$$

where $\lambda_i = V_{id}^* V_{ib}^*$, $x_i = m_i^2/M_W^2$, and

$$\mathcal{O}^{\Delta S=2} = \bar{d}\gamma_\mu L s \bar{d} \gamma^\mu L s \qquad (15)$$

The functions $\bar{E}(x)$, $E(x, x')$ are explicitly given by [4]

$$\bar{E}(x) = -\frac{3x^3 \ln x}{2(x-1)^3} - \frac{x(x^2 - 11x + 4)}{4(x-1)^2} \qquad (16)$$

$$E(x,x') = -x,x' \left\{ \frac{1}{x - x'} \left[\frac{(x^2 - 8x + 4)\ln x}{4(x-1)^2} + (x \leftrightarrow x') \right] - \frac{3}{4(x-1)(x'-1)} \right\} \qquad (17)$$

These functions have the following properties:

$$\bar{E}(x) \approx -x \quad \text{for} << 1, \qquad (18)$$

$$E(x, x') \approx x' \ln x'/x, \quad \text{for } x' << x << 1$$
$$\approx x' \ln x', \quad \text{for } x' << x \approx 1 \qquad (19)$$

Taking the u, c and t quark masses as

$$m_u = 0.0056 \text{ GeV}, \; m_c = 1.35 \text{ GeV}, \; m_t = 174 \text{ GeV} \qquad (20)$$

gives

$$\bar{E}(x_u) = -4.87\times10^{-9},\ \bar{E}(x_c) = -2.83\times10^{-4},$$
$$\bar{E}(x_t) = +2.15$$
$$E(x_u, x_c) = -5.35\times10^{-8},\ E(x_u, x_t) = -9.33\times10^{-8},$$
$$E(x_t, x_t) = -2.31\times10^{-3} \quad (21)$$

In the subsequent sections, K^o–$\bar{K}^o$, B^o–$\bar{B}^o$ and B_s^o–$\bar{B}_s^o$ mixings will be considered separately.

K^o–$\bar{K}^o$ SYSTEM

For the K^o–$\bar{K}^o$ system, we have

$$\Delta\Gamma \approx -2\Delta m \quad (22)$$

to within 5% accuracy. The indirect CP violation parameter ε is then given by

$$\varepsilon \approx \frac{e^{i\pi/4}\,\mathrm{Im}\,M_{12}}{2\sqrt{2}\,\mathrm{Re}\,M_{12}} \quad (23)$$

Assuming that M_{12} is due entirely to the box diagrams, we can then use Eq.(14) to give an estimate of ε. Now for K^o–$\bar{K}^o$ system,

$$|\lambda_u^2| \sim 0.05,\quad |\lambda_c^2| \sim 0.05,\quad |\lambda_t^2| \sim 1.6\times10^{-7} \quad (24)$$

so that

$$|\lambda_c^2\,\bar{E}(x_c)| \sim 1.4\times10^{-5} \quad (25)$$

is the dominant term in Eq.(14). The other terms are at best of order 10^{-7}. the ratio of Im M_{12} to Re M_{12} is then given purely by the K-M matrix elements:

$$\frac{\mathrm{Im}\,M_{12}}{\mathrm{Re}\,M_{12}} = \frac{\mathrm{Im}\,\lambda_c^2}{\mathrm{Re}\,\lambda_c^2} = \frac{\mathrm{Im}(V_{cd}^*\,V_{cs})^2}{\mathrm{Re}(V_{cd}^*\,V_{cs})^2} = \tan 2\phi, \quad (26)$$

where ϕ is the phase of V_{cs} relative to V_{cd}. This gives the indirect CP violation parameter $\varepsilon(K)$ for K^o–$\bar{K}^o$ system as

$$|\varepsilon(K)| \approx \frac{1}{2\sqrt{2}}\tan 2\phi \quad (27)$$

Since

$$|\varepsilon(K)| \approx (2.266 \pm 0.017)\times10^{-3} \quad (28)$$

we find

$$\tan\phi = 3.2\times10^{-3} \quad (29)$$

B^o–$\bar{B}^o$ SYSTEM

For the B^o–$\bar{B}^o$ system,

$$\Delta\Gamma \ll \Delta m \quad (30)$$

so that the analogous CP violation parameter $\varepsilon(B)$ due to mixing is given by

$$\varepsilon(B) = \frac{i}{2}(\mathrm{Im}M_{12}/\mathrm{Re}\,M_{12}) \quad (31)$$

where M_{12} here denotes the dispensive part of the off-diagonal B^o–$\bar{B}^o$ mass matrix. The effective Hamiltonian for B^o–$\bar{B}^o$ mixing is given by an expression similar to Eq.(14). But here $\lambda_i = V_{id}^*\,V_{ib}$, and

$$\mathcal{O}^{\Delta B=2} = \bar{d}\gamma_\mu L b \bar{d}\gamma^\mu L b \quad (32)$$

For the B^o–$\bar{B}^o$ system, we have

$$|\lambda_u^2| \sim 10^{-5},\quad |\lambda_c^2| \sim 10^{-4},\quad |\lambda_t^2| \sim 10^{-4}, \quad (33)$$

so that

$$|\lambda_t^2\,\bar{E}(x_t)| \sim 2.2\times10^{-4} \quad (34)$$

is the dominant contribution. In comparison, the other terms are of order 10^{-7} or smaller. This gives

$$\varepsilon(B) \approx \frac{1}{2}\frac{\mathrm{Im}\,\lambda_t^2}{\mathrm{Re}\,\lambda_t^2} = \frac{1}{2}\frac{\mathrm{Im}(V_{td}^*\,V_{tb})^2}{\mathrm{Re}(V_{td}^*\,V_{tb})^2} = \frac{1}{2}\tan 2\phi' \quad (35)$$

where ϕ' is the phase of V_{tb} relative to V_{td}.

B_s^o–$\bar{B}_s^o$ SYSTEM

For the B^o–$\bar{B}^o$ system, as in the B^o–$\bar{B}^o$ system, we have $\Delta\Gamma \ll \Delta m$, so that the CP violation parameter $\varepsilon(B_s)$ is also given by Eq.(31). The B^o–$\bar{B}^o$ mixing is given by Eq.(14) but ith $\lambda_i = V_{is}^*\,V_{ib}$ and an expression for the operator $\mathcal{O}$ analogous to Eq.(32).

For this system, we have

$$|\lambda_u^2| \sim 4.4\times10^{-7},\ |\lambda_c^2| \sim 1.6\times10^{-3},\ |\lambda_t^2| \sim 1.6\times10^{-3} \quad (36)$$

Again the term

$$\left|\lambda_t^2\, \bar{E}(x_t)\right| \sim 3.4 \times 10^{-3} \tag{37}$$

is dominant in contribution. Other terms are much smaller, of order 10^{-9} or less. The CP violation parameter is thus given by

$$\left|\varepsilon(B_s)\right| = \frac{1}{2}\,\frac{\mathrm{Im}(V_{ts}^{*}\,V_{tb})^2}{\mathrm{Re}(V_{ts}^{*}\,V_{tb})^2} = \frac{1}{2}\tan 2\,\phi'' \tag{38}$$

where ϕ'' is the phase of V_{tb} relative to V_{ts}.

CONCLUSION

The dispersive part of M_{12} of the off-diagonal particle-antiparticle mass matrix is calculated from the box diagrams of Fig.1 within the framework of the Standard Model. A knowledge of the ratio $\Delta\Gamma/\Delta m$ allows us to express the indirect CP violation parameter ε arising from such a particle-antiparticle mixing in terms of the relative phases of the K-M matrix elements.

In the calculation, I have assumed that the box diagrams provide the dominant contributions to particle-antiparticle mixing. This is a sound assumption for B^o–$\bar{B}^o$ and B_s^o–$\bar{B}_s^o$ systems [5]. But for the K^o–$\bar{K}^o$ system, contributions from the box diagrams, the so-called short-distance contributions, are not the only important contributions. Long-distance contributions may be important [5]. Taking into account the long-distance contributions to M_{12}, which are predominantly real, Eq.(27) for the K^o–$\bar{K}^o$ system is replaced by

$$\left|\varepsilon(K)\right| \approx \frac{1}{2\sqrt{2}}\,\frac{\tan 2\phi}{1+r} \tag{39}$$

where

$$r = \mathrm{Re}M_{12}^{ld}/\mathrm{Re}M_{12}^{sd} \tag{40}$$

Here ld and sd stand for long-distance and short-distance contributions respectively. Calculation of r is, however, very much model dependent.

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