CP violation arising from particle-antiparticle mixing

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ABSTRACT Indirect CP violation arising from particle-antiparticle mixing is calculated from the box diagrams in the Standard Model for $K^0-\bar{K}^0$, $B^0-\bar{B}^0$ and $B^+_s-\bar{B}^+_s$ systems. The CP violation parameter for each of the systems is shown to be closely related to the relative phases of the Kobayashi-Maskawa matrix elements.

ABSTRAK Perlanggaran CP taklangsung yang berhasil dari percampuran zarah-antizarah dikira dari rajah-rajah kotak di dalam Model Piawai untuk sistem-sistem $K^0-\bar{K}^0$, $B^0-\bar{B}^0$ dan $B^+_s-\bar{B}^+_s$. Parameter perlanggaran CP bagi setiap sistem itu ditunjukkan berhubung rapat dengan fasa-fasa relatif di antara unsur-unsur matriks Kobayashi-Maskawa.

(CP violation, particle-antiparticle mixing, Standard Model, Kobayashi-Maskawa matrix)

INTRODUCTION

Large particle-antiparticle mixing is observed in $K^0-\bar{K}^0$, $B^0-\bar{B}^0$ and $B^+_s-\bar{B}^+_s$ systems. In the case of $K^0-\bar{K}^0$ system, such a mixing gives rise to two distinct mass eigenstates, $K^0_L$ and $K^0_S$, with decay lifetimes of 0.89310$^{-10}$ s and 5.1710$^{-4}$ s respectively, and a mass difference of [1]

$$\Delta m(K) = m(K_L) - m(K_S) = 3.51 \times 10^{-12} \text{ MeV}$$  \hspace{1cm} (1)

Mixing in the $B^0-\bar{B}^0$ system is measured by the mixing parameter $\chi(B)$ [1]

$$\chi(B) = \frac{\Gamma(B \rightarrow \mu^- X) \Gamma(B \rightarrow \mu^+ X)}{\Gamma(B \rightarrow X)} = 0.156 \pm 0.024$$  \hspace{1cm} (2)

The two mass eigenstates, $B^0_{L}$ and $B^0_{S}$, have a mass difference of [1]

$$\Delta m(B) = (3.4 \pm 0.4) \text{ MeV}$$  \hspace{1cm} (3)

but do not have noticeably distinct decay lifetimes.

The $B^0-\bar{B}^0$ system is also observed to have large mixing, with a mixing parameter of [1]

$$\chi(B) = 0.62 \pm 0.13$$  \hspace{1cm} (4)

The two mass eigenstates arising from mixing have a mass difference of [1]

$$\Delta m(B_s) > 1.2 \times 10^{-9} \text{ MeV}$$  \hspace{1cm} (5)

but, again, do not differ noticeably in the lifetimes.

Within the Standard Model, particle-antiparticle mixing arises from higher order weak interactions, the main contributions of which come from the box diagrams of Fig.1 [2]. Depicted in Fig.1 are the Feynman diagrams that give rise to $K^0-\bar{K}^0$ mixing. Feynman diagrams that contribute to $B^0-\bar{B}^0$ mixing and $B^+_s-\bar{B}^+_s$ mixing are obtained by replacing respectively the external $s$-quark and $d$-quark by the $b$-quark. The internal quark lines, $i$ and $j$, can be a $u$, $c$ or $t$-quark.

Figure 1. Box diagrams within the Standard Model that give rise to $K^0-\bar{K}^0$ mixing. The internal quark lines $i$, $j$, can be a $u$, $c$ or $t$ quark. $B^0-\bar{B}^0$ ($B^+_s-\bar{B}^+_s$) mixing is described by similar diagrams with the external $s$ ($d$) quark lines replaced by $b$ quark.

Weak interactions of the quarks are described, in the Standard Model, by the following Lagrangian:

$$\mathcal{L}_{\text{(quark)}} = \frac{ie}{\sqrt{2}}(\bar{u}, \bar{c}, \bar{t})\gamma^\mu LV \begin{pmatrix} d \\ b \end{pmatrix} W_{\mu} + c.c. \hspace{1cm} (6)$$

where $V$ is the Kobayashi-Maskawa (K-M) mixing matrix [3]. The magnitudes of the K-M matrix elements are approximately given by

$$|V| \approx \begin{pmatrix} 1 & 0.22 & 0.003 \\ 0.22 & 1 & 0.04 \\ 0.01 & 0.04 & 1 \end{pmatrix} \hspace{1cm} (7)$$
For three families of quarks, the K-M matrix contains a complex phase which gives rise to CP violation effects in a natural way. Because of this complex phase, the box diagrams of Fig.1 provide a definite connection between particle-antiparticle mixing and CP violation in such a system, the so-called indirect CP violation.

In this paper, I shall exploit the box diagrams to derive definite relationship between relative phase among the different K-M matrix on the one-hand, and the CP violation parameter on the other.

**DESCRIPTION OF CP VIOLATION**

In this section, I shall make specific reference to $K^0 - \bar{K}^0$ mixing as a generic case for the three particle-antiparticle systems. The box diagrams of Fig.1 give rise to $\Delta S = 2$ effective Hamiltonian $\mathcal{H}(\Delta S = 2)$, and hence to off-diagonal element of the $K^0 - \bar{K}^0$ mass matrix

$$\langle K^0 | \mathcal{H}(\Delta S = 2) | \bar{K}^0 \rangle = M_{12} - i \Gamma_{12} / 2 \quad (8)$$

where $M_{12}$ and $\Gamma_{12}$ are respectively the dispersive and absorptive parts of the off-diagonal element of the mass matrix.

Diagonalizing the mass matrix gives two distinct mass eigenstates, which can be written in the following form:

$$| K_{L,S} \rangle = (2(1 + |\varepsilon|^2))^{-1/2} [(1 + \varepsilon) | K^0 \rangle + (1 - \varepsilon) | \bar{K}^0 \rangle] \quad (9)$$

where $\varepsilon$ is the indirect CP violation parameter.

As CP violation is a small effect, we have

$$\text{Im} M_{12} \ll \text{Re} M_{12}, \quad \text{Im} \Gamma_{12} \ll \text{Re} \Gamma_{12}, \quad \text{Im} \Gamma_{12} \ll \text{Im} M_{12} \quad (10)$$

This greatly simplifies the expressions for $K_L - K_S$ mass difference $\Delta m$, their decay rate difference $\Delta \Gamma$, and the indirect CP violation parameter $\varepsilon$:

$$\Delta m = \text{Re} M_{12} \quad (11)$$

$$\Delta \Gamma = 2 \text{Re} \Gamma_{12} \quad (12)$$

$$\varepsilon = (i/2) \frac{\text{Im} M_{12} - i \text{Im} \Gamma_{12} / 2}{\text{Re} M_{12} - i \text{Re} \Gamma_{12} / 2} \quad (13)$$

In the next section, I shall give the explicit result for the dispersive part, $M_{12}'$, of the off-diagonal mass matrix element from the box diagrams. The absorptive part, $\Gamma_{12}'$, will be deduced from a knowledge of $\Delta \Gamma$ and $\Delta m$.

**EXPLICIT RESULT FROM THE BOX DIAGRAMS**

The calculation of $M_{12}'$ from the box diagrams is straightforward. A detailed calculation gives

$$\mathcal{H}_{\text{eff}} = \frac{G_F^2 M_W^2}{4\pi^2} \oint \mathcal{A} \{ \lambda_1^2 E(x_0) + \lambda_2^2 E(x_2) + \lambda_3^2 E(x_3) 
+ \lambda_1 \lambda_2 E(x_0, x_2) + \lambda_1 \lambda_3 E(x_0, x_2) + \lambda_2 \lambda_3 E(x_0, x_2) \} \quad (14)$$

where $\lambda_i = V_{ij}^2 V_{ji}^0$, $x_i = m_i^2 / M_W^2$, and

$$\oint \mathcal{A} \mathcal{S} = \bar{d} \gamma_\mu L d \gamma^\mu L s \quad (15)$$

The functions $\bar{E}(x), E(x, x')$ are explicitly given by [4]

$$\bar{E}(x) = \frac{-3x^3 \ln x}{2(x-1)^3} - \frac{x(x^2 - 11x + 4)}{4(x-1)^4} \quad (16)$$

$$E(x, x') = -x \ln \left[ \frac{1}{x-x'} \left( \frac{(x^2 - 8x + 4)\ln x}{4(x-1)^2} + \left( x \leftrightarrow x' \right) \right) - \frac{3}{4(x-1)(x'-1)} \right] \quad (17)$$

These functions have the following properties:

$$\bar{E}(x) = -x \quad \text{for } x < 1, \quad (18)$$

$$E(x, x') = x' \ln x / x, \quad \text{for } x' < x < 1$$

$$= x' \ln x', \quad \text{for } x' = x = 1 \quad (19)$$

Taking the $u, c$ and $t$ quark masses as

$$m_u = 0.0056 \text{ GeV}, \quad m_c = 1.35 \text{ GeV}, \quad m_t = 174 \text{ GeV} \quad (20)$$

gives
\( \overline{E}(x_u) = -4.87 \times 10^{-9}, \overline{E}(x_c) = -2.83 \times 10^{-4}, \overline{E}(x_t) = +2.15 \)

\( E(x_u, x_c) = -5.35 \times 10^{-8}, E(x_u, x_t) = -9.33 \times 10^{-8}, \) 

\( E(x_t, x_t) = -2.31 \times 10^{-3} \) \( (21) \)

In the subsequent sections, \( K^0 - \overline{K}^0, B^0 - \overline{B}^0 \) and \( B^+ - \overline{B}^0 \) mixings will be considered separately.

\( K^0 - \overline{K}^0 \) SYSTEM

For the \( K^0 - \overline{K}^0 \) system, we have

\[ \Delta \Gamma = -2 \Delta m \] \( (22) \)

to within 5% accuracy. The indirect CP violation parameter \( \varepsilon \) is then given by

\[ \varepsilon = \frac{e^{\text{mix}} \text{Im} M_{12}}{2 \sqrt{2} \text{Re} M_{12}} \] \( (23) \)

Assuming that \( M_{12} \) is due entirely to the box diagrams, we can then use Eq.(14) to give an estimate of \( \varepsilon \). Now for \( K^0 - \overline{K}^0 \) system,

\[ |\lambda_a^2| \approx 0.05, \quad |\lambda_b^2| \approx 0.05, \quad |\lambda_c^2| \approx 1.6 \times 10^{-7} \] \( (24) \)

so that

\[ |\lambda_c^2 \overline{E}(x_j)| \approx 1.4 \times 10^{-5} \] \( (25) \)

is the dominant term in Eq.(14). The other terms are at best of order \( 10^{-7} \). The ratio of \( \text{Im} M_{12} \) to \( \text{Re} M_{12} \) is then given purely by the K-M matrix elements:

\[ \frac{\text{Im} M_{12}}{\text{Re} M_{12}} = \frac{\text{Im} \lambda_a^2}{\text{Re} \lambda_a^2} = \frac{\text{Im} (V_{ud}^* V_{cd})^2}{\text{Re} (V_{ud}^* V_{cd})^2} = \tan 2 \phi, \] \( (26) \)

where \( \phi \) is the phase of \( V_{cd}^* \) relative to \( V_{ud}^* \). This gives the indirect CP violation parameter \( \varepsilon(K) \) for \( K^0 - \overline{K}^0 \) system as

\[ |\varepsilon(K)| = \frac{1}{2 \sqrt{2}} \tan 2 \phi \] \( (27) \)

Since

\[ |\varepsilon(K)| = (2.266 \pm 0.017) \times 10^{-3} \] \( (28) \)

we find

\[ \tan \phi = 3.2 \times 10^{-3} \] \( (29) \)

\( B^0 - \overline{B}^0 \) SYSTEM

For the \( B^0 - \overline{B}^0 \) system,

\[ \Delta \Gamma \approx \Delta m \] \( (30) \)

so that the analogous CP violation parameter \( \varepsilon(B) \) due to mixing is given by

\[ \varepsilon(B) = \frac{i}{2} \left( \text{Im} M_{12} / \text{Re} M_{12} \right) \] \( (31) \)

where \( M_{12} \) here denotes the dispersive part of the off-diagonal \( B^0 - \overline{B}^0 \) mass matrix. The effective Hamiltonian for \( B^0 - \overline{B}^0 \) mixing is given by an expression similar to Eq.(14). But here \( \lambda_i = V_{ub}^* V_{ub} \), and

\[ \mathbf{B} \Delta \theta^2 = \overline{d}_\gamma \overline{U}_b \overline{d}_\gamma \overline{U}_b \overline{d}_\gamma \overline{U}_b \overline{d}_\gamma \overline{U}_b \] \( (32) \)

For the \( B^0 - \overline{B}^0 \) system, we have

\[ |\lambda_a^2| \approx 10^{-5}, \quad |\lambda_b^2| \approx 10^{-4}, \quad |\lambda_c^2| \approx 10^{-4}, \] \( (33) \)

so that

\[ |\lambda_c^2 \overline{E}(x)j| \approx 2.2 \times 10^{-4} \] \( (34) \)

is the dominant contribution. In comparison, the other terms are of order \( 10^{-5} \) or smaller. This gives

\[ \varepsilon(B) = \frac{1}{2} \frac{\text{Im} \lambda_b^2}{\text{Re} \lambda_b^2} = \frac{1}{2} \frac{\text{Im} (V_{ud}^* V_{ub})^2}{\text{Re} (V_{ud}^* V_{ub})^2} = \tan 2 \phi' \] \( (35) \)

where \( \phi' \) is the phase of \( V_{ub}^* \) relative to \( V_{ud}^* \).

\( B^+ - \overline{B}^0 \) SYSTEM

For the \( B^+ - \overline{B}^0 \) system, as in the \( B^0 - \overline{B}^0 \) system, we have \( \Delta \Gamma \approx \Delta m \), so that the CP violation parameter \( \varepsilon(B) \) is also given by Eq.(31). The \( B^+ - \overline{B}^0 \) mixing is given by Eq.(14) but ith \( \lambda_i = V_{ub}^* V_{ub} \) and an expression for the operator \( \mathbf{B} \) analogous to Eq.(32).

For this system, we have

\[ |\lambda_a^2| \approx 4.4 \times 10^{-7}, |\lambda_b^2| \approx 1.6 \times 10^{-3}, |\lambda_c^2| \approx 1.6 \times 10^{-3} \] \( (36) \)
Again the term

$$|\lambda_{t}^{2} \bar{E}(x)| = 3.4 \times 10^{-3}$$

(37)

is dominant in contribution. Other terms are much smaller, of order $10^{-9}$ or less. The CP violation parameter is thus given by

$$|\varepsilon(B_{d})| = \frac{1}{2} \frac{\text{Im}(V_{ud}^{*} V_{cb})^{2}}{\text{Re}(V_{ud}^{*} V_{cb})^{2}} = \frac{1}{2} \tan 2 \phi^{\prime\prime}$$

(38)

where $\phi^{\prime\prime}$ is the phase of $V_{ub}$ relative to $V_{ts}$.

CONCLUSION

The dispersive part of $M_{d2}$ of the off-diagonal particle-antiparticle mass matrix is calculated from the box diagrams of Fig.1 within the framework of the Standard Model. A knowledge of the ratio $\Delta N/\Delta m$ allows us to express the indirect CP violation parameter $\varepsilon$ arising from such a particle-antiparticle mixing in terms of the relative phases of the K-M matrix elements.

In the calculation, I have assumed that the box diagrams provide the dominant contributions to particle-antiparticle mixing. This is a sound assumption for $B^{0} - \bar{B}^{0}$ and $B^{-} - \bar{B}^{+}$ systems [5]. But for the $K^{0} - \bar{K}^{0}$ system, contributions from the box diagrams, the so-called short-distance contributions, are not the only important contributions. Long-distance contributions may be important [5]. Taking into account the long-distance contributions to $M_{12}$, which are predominantly real, Eq.(27) for the $K^{0} - \bar{K}^{0}$ system is replaced by

$$|\varepsilon(K)| = \frac{1}{2 \sqrt{2}} \frac{\text{tan } 2\phi}{1 + r}$$

(39)

where

$$r = \frac{\text{Re} M_{12}^{sd}}{\text{Re} M_{12}^{sd}}$$

(40)

Here $ld$ and $sd$ stand for long-distance and short-distance contributions respectively. Calculation of $r$ is, however, very much model dependent.

REFERENCES